Differentiation Rules:

Recall: Sum / Difference Rule: $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

Power Rule: $\frac{d}{dx}\left(x^n\right) = n\, x^{n-1}$

Constant Multiple Rule: $\frac{d}{dx}\left(c\, f(x)\right) = c \cdot \frac{d}{dx} f(x)$

⊛ $\frac{d}{dx}(x) = 1$, $\frac{d}{dx}(\text{any constant}) = 0$

Eg. $h(x) = 5\sqrt{x^3} - 2\sqrt[3]{x^{11}}$. Find $h'(x)$.

$$h(x) = 5x^{3/2} - 2x^{\frac{11}{3}}$$

So $h'(x) = 5 \cdot \frac{3}{2} x^{\frac{3}{2}-1} - 2 \cdot \frac{11}{3} x^{\frac{11}{3}-1}$

$$= \frac{15}{2} x^{1/2} - \frac{22}{3} x^{\frac{8}{3}}$$

Eg. $g(x) = \frac{(x^3+4)(x^2+1)}{x}$

$$= \frac{x^3 \cdot x^2 + x^3 \cdot 1 + 4 \cdot x^2 + 4 \cdot 1}{x}$$

$$= \frac{x^5 + x^3 + 4x^2 + 4}{x}$$

$$= x^4 + x^2 + 4x + \frac{4}{x}$$

Then $g'(x) = \frac{d}{dx}\left(x^4 + x^2 + 4x + \frac{4}{x}\right)$

$$= 4x^3 + 2x + 4 \cdot 1 + 4 \frac{d}{dx}\left(x^{-1}\right)$$

$$= 4x^3 + 2x + 4 + 4(-1)x^{-1-1}$$

$$= 4x^3 + 2x + 4 - \frac{4}{x^2}$$

Product Rule:

Let $A(x)=$ product of $f(x)$ & $g(x)$

$$= f(x)\cdot g(x)$$

Then, $A(x+h) = f(x+h)\cdot g(x+h)$

So, $A(x+h) - A(x)$

$$= f(x+h)\cdot g(x+h) - f(x)\cdot g(x)$$

$$= f(x+h)\cdot g(x+h) - f(x)\cdot g(x+h) + f(x)\cdot g(x+h) - f(x)\cdot g(x)$$

$$= [f(x+h)-f(x)]\,g(x+h) + f(x)[g(x+h)-g(x)]$$

So, $$\frac{A(x+h)-A(x)}{h} = \frac{[f(x+h)-f(x)]\,g(x+h) + f(x)[g(x+h)-g(x)]}{h}$$

$$= \frac{[f(x+h)-f(x)]\,g(x+h)}{h} + \frac{f(x)[g(x+h)-g(x)]}{h}$$

Then, $$\frac{d}{dx}(A(x)) = \lim_{h\to 0}\frac{A(x+h)-A(x)}{h}$$

$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to 0}\left\{\frac{[f(x+h)-f(x)]\,g(x+h)}{h} + \frac{f(x)[g(x+h)-g(x)]}{h}\right\}$$

$$= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\cdot g(x+h)\right] + \lim_{h\to 0}\left[f(x)\cdot\frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\cdot\lim_{h\to 0} g(x+h) + f(x)\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$= f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\, f'(x) - g'(x) \cdot f(x)}{[g(x)]^2}$$

⊛ For Remembering : $\dfrac{\text{lo d hi} - \text{hi d lo}}{\text{lo lo}}$